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Fluctuations on liquid–gas interfaces: cut-off limit problem of the surface waves

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Abstract. The influence of the stress tensor fluctuations in the bulk on a liquid–gas interface is studied, using the generalized Langevin equation formalism and the new fact (recently shown by the present authors) that these fluctuations are spatially correlated because of the viscous–dissipative interactions. In this way, finite surface wave characteristics such as the root mean square amplitude and energy density are established. In the literature the surface wave spatial correlations are treated by assuming an upper limit of wavevectors called a cut-off vector. It follows from the present theory that the cut-off wavevector is directly related to the dissipative correlation length, being of the order of the mean free path of molecules.

1. Introduction

In the present paper the behaviour of the transverse fluctuating waves on a liquid–gas interface produced by the stress tensor fluctuations in the bulk is described. The relation between the wave profile $\zeta(\mathbf{r}, t)$ ($\mathbf{r} \equiv (x, y)$) and the normal projection on the surface of the total stress tensor fluctuations $\Delta\mathbf{P} = \mathbf{P} - \langle\mathbf{P}\rangle_\zeta$ has the following standard Fourier form:

$$\zeta(\mathbf{k}, \omega) = \alpha(\mathbf{k}, \omega)\Delta P(\mathbf{k}, \omega) \quad (1)$$

where $\alpha(\mathbf{k}, \omega)$ is the linear susceptibility function. Knowledge of α and the statistical properties of ΔP provides the possibility of performing a correlation analysis of the amplitudes ζ and their velocities. The stress tensor $\langle\mathbf{P}\rangle_\zeta$ consists of the equilibrium stress tensor $\langle\mathbf{P}\rangle$ of the fluid and the perturbation caused by the surface wave motion. It represents the mean value of the total stress tensor \mathbf{P} averaged along the coordinates and pulses of the fluid particles by the restriction of a constant surface profile and is due to the random nature of the surface waves; it is also a random quantity with a mean value $\langle\mathbf{P}\rangle$. Since the mean value of ΔP is zero, it follows from (1) that $\langle\zeta\rangle = 0$; hence essential information about the behaviour of the surface waves has to be searched for in the higher statistical moments of ζ , such as the autocorrelation function

$$\langle\zeta(\mathbf{k}, \omega)\zeta(\mathbf{k}_1, \omega_1)\rangle = \alpha(\mathbf{k}, \omega)\alpha(\mathbf{k}_1, \omega_1)\langle\Delta P(\mathbf{k}, \omega)\Delta P(\mathbf{k}_1, \omega_1)\rangle. \quad (2)$$

The equilibrium fluctuations ΔA of the physical parameters $A(\mathbf{r}, t)$ (in the present case, $A \equiv \zeta$ or P) are stationary and uniform stochastic processes [1, 2], which justifies the presentation of their autocorrelation functions in the form

$$\langle A(\mathbf{k}, \omega)A(\mathbf{k}_1, \omega_1)\rangle = C_{AA}(\mathbf{k}, \omega)\delta(\mathbf{k} + \mathbf{k}_1)\delta(\omega + \omega_1) \quad (3)$$

where $C_{AA}(k, \omega) = (1/\pi) \int_0^\infty \int_0^\infty \langle A(\mathbf{r}, t) A(\mathbf{r} + \mathbf{R}, t + \tau) \rangle \cos(\omega\tau) J_0(kR) R dR d\tau$ is the corresponding spectral density with $J_0(\cdot)$ being a Bessel function of the first kind of zero order. Also owing to the surface isotropy, the linear admittance function α depends only on the modulus of the wavevector [3, 4]: $\alpha = \alpha(k, \omega)$. Thus, from (2) and (3), a link between the spectral densities $C_{\xi\xi}(k, \omega)$ and $C_{PP}(k, \omega)$ is obtained:

$$C_{\xi\xi}(k, \omega) = \alpha(k, \omega)\alpha(-k, -\omega)C_{PP}(k, \omega). \quad (4)$$

From this relation, by knowledge of $\alpha(k, \omega)$ or $C_{PP}(k, \omega)$ and using the fluctuation-dissipation theorem (FDT), a number of correlation characteristics of the fluctuating capillary-gravity surface waves can be derived. The main aim of the present paper is to show that viscous-dissipative correlation of the stress tensor fluctuations recently established by the present authors [7, 8] leads to a natural upper wavevector of the spatial spectral density of the fluctuating surface waves and to reasonable values for many of their mean characteristics.

2. Fluctuation-dissipation theorem

The first step in the concretization of (4) is the application of the so-called second FDT connecting the dissipative and fluctuating forces in such a way that their coaction determines the tendency of the system towards a thermodynamic equilibrium state. The second FDT can be formulated by introducing a generalized Langevin equation (GLE) [1, 5, 6], governing the evolution of an arbitrary physical quantity $A(t)$ of the system:

$$\frac{dA}{dt} + \int_0^t \frac{\langle F_f(t) F_f(t_1) \rangle}{\langle A^2 \rangle} A(t_1) dt_1 = F_e(t) + F_f(t). \quad (5)$$

According to this equation the evolution of A is the result of the coaction of the external force F_e and the fluctuating force F_f ($\langle F_f \rangle = 0$) related to A . The integral in (5) plays the role of a dissipative force F_d (viscous, ohmic, etc, resistance) and its kernel represents a quantitative expression for the second FDT. In the case of fluctuating capillary-gravity waves considered, the role of the fluctuating force will be played by the normal projection ΔP of the total stress tensor fluctuations on the interface.

A necessary condition for performing the statistical analysis by the GLE (5) is knowledge of a phenomenological admittance function between the mean value of A and F_e , which determines the autocorrelation function of F_f . In the general case, the admittance function for capillary-gravity surface waves is rather complex [4]:

$$\alpha(k, \omega) = (k/\rho) / [\omega_0^2 + (i\omega + 2\nu k^2)^2 - 4(\nu k^2)^2 \sqrt{1 + i\omega/\nu k^2}].$$

This is not suitable for the description of the behaviour of fluctuating waves by the GLE (5). Here $\omega_0^2 = \gamma k^3/\rho + gk$, γ is the surface tension, ρ and ν are the mass density and kinematic viscosity, respectively, of the liquid and g is the acceleration due to gravity. For simplicity and physical transparency the so-called high-viscosity approximation ($\omega \ll \nu k^2$) for the admittance function $\alpha(k, \omega)$ will be applied [3, 4]:

$$\alpha^0(k, \omega) = (k/\rho) / (\omega_0^2 - \omega^2 + 2i\omega\nu k^2). \quad (6)$$

As the main subject of interest in the present paper is to describe the upper cut-off limit of surface waves, the use of the high-viscosity approximation corresponding to high wavenumbers k is physically justified.

The susceptibility α^0 is equivalent to an evolution equation of the spatial Fourier components of the surface wave amplitude:

$$(\rho/k)[\partial^2\zeta(\mathbf{k}, t)/\partial t^2] + 2\rho\nu k[\partial\zeta(\mathbf{k}, t)/\partial t] = -(\gamma k^2 + \rho g)\zeta(\mathbf{k}, t) + \Delta P(\mathbf{k}, t) \tag{7}$$

which compared with the GLE (5) leads to the following identifications: $A \equiv (\rho/k)[\partial\zeta(\mathbf{k}, t)/\partial t]$, $F_e \equiv -(\gamma k^2 + \rho g)\zeta(\mathbf{k}, t)$ and $F_f \equiv \Delta P(\mathbf{k}, t)$. Because of the simple type of (7), the juxtaposition of the forces F_e , F_f and F_d is obvious. For instance the viscous friction in equation (7) is described by the second term $2\rho\nu k[\partial\zeta(\mathbf{k}, t)/\partial t]$ and for this reason it corresponds to the dissipative force F_d :

$$2\rho\nu k \frac{\partial\zeta(\mathbf{k}, t)}{\partial t} = \int_0^t \frac{\langle \Delta P(\mathbf{k}, t) \Delta P(\mathbf{k}, t_1) \rangle}{(\rho/k)[\partial\zeta(\mathbf{k}, t)/\partial t]^2} \frac{\partial\zeta(\mathbf{k}, t_1)}{\partial t_1} dt_1.$$

This last equation leads to a relation for the spectral density of the autocorrelation function of the normal projection of the stress tensor fluctuations, representing a quantitative expression for the second FDT:

$$C_{PP}^0(k, \omega) = \frac{2}{\pi} \nu \rho^2 \int_{-\infty}^{\infty} \omega^2 C_{\zeta\zeta}(k, \omega) d\omega \equiv C_{PP}^0(k). \tag{8}$$

The superscript zero means that $C_{PP}^0(k)$ describes the fluctuations ΔP with accuracy corresponding to that of α^0 . The next step in the further analysis is to present with the same accuracy the important relation (4) between the spectral densities C_{PP} and $C_{\zeta\zeta}$.

3. Stress tensor fluctuations

The ω -independence of the spectral density $C_{PP}^0(k)$ is due to the approximate description of the susceptibility α by α^0 . Since α^0 is obtained from the exact admittance function in the limit $\omega \rightarrow 0$, $C_{PP}^0(k)$ is equal to $C_{PP}(k, \omega = 0)$. Thus equation (4) is

$$C_{\zeta\zeta}(k, \omega) = \alpha^0(k, \omega)\alpha^0(-k, -\omega)C_{PP}(k, 0). \tag{9}$$

As seen from (9), to obtain the autocorrelation function of the fluctuating capillary-gravity wave amplitudes, knowledge of the spectral density $C_{PP}(k, 0)$ is required.

In a previous paper [7, 8] the present authors have shown that the spectral density of the normal projection of the stress tensor fluctuations has the form $C_{PP}(kl_c, \omega\tau_c)$, where $l_c \sim \nu/c$ and $\tau_c \sim \nu/c^2$ are the viscous-dissipative correlation length and time, c being the thermal velocity of the liquid particles. The same papers also lead to the conclusion that the spectral density $C_{PP}(k, 0)$ could be expressed as

$$C_{PP}(k, 0) = (1/\pi)^2 k_B T \rho \nu k f(kl_c) \tag{10}$$

where $k_B T$ is the Boltzmann factor and the unknown function $f(\cdot)$ has the asymptotic properties $f(0) = 1$ and $f(\infty) = 0$. Equation (10) is the same as that proposed by the present authors from more general considerations [8]. Neglecting the viscous-dissipative correlations ($c \rightarrow \infty$ at constant ν) leads to the well known classical result $C_{PP}^0(k) = (1/\pi)^2 k_B T \rho \nu k$ [9].

The physical nature of the viscous-dissipative correlation as a self-organization structure could be elucidated by means of the laws of local turbulence. In 1941, Oboukhov and Kolmogorov (cf [2, 10]) established a characteristic length L of the small-scale turbulence, below which the motion is laminar. The size of these turbulent subdomains obeys the relation $L^4 \sim \nu^3/w$, where w is the local power (per unit mass) of the dissipation of energy as viscous friction. Because of stationarity, the power w has to be equal to that of the external sources. In the description of fluctuations the thermal molecular movement plays the role of external source and w could be estimated as $w \sim \epsilon/\tau$, ϵ and τ being the specific energy of the thermal motion and the excitation time of the fluctuations, respectively. Because of stationarity, the characteristic time τ of the source has to be equal to the mean relaxation time L^2/ν of the energy dissipation. From this viewpoint the Oboukhov-Kolmogorov law has an alternative expression as $\epsilon \sim (\nu/L)^2$. Since the specific energy of the thermal motion of molecules is proportional to their mean square velocity, i.e. $\epsilon \sim c^2$, the size and lifetime of the fluctuations are estimated from this law as $L \sim \nu/c$ and $\tau \sim \nu/c^2$, i.e. they are identical with the correlation length l_c and time τ_c introduced above.

4. Fluctuating capillary-gravity waves

The main result obtained by combination of (6), (9) and (10) is the expression for the spectral density $C_{\xi\xi}(k, \omega)$ of the autocorrelation function of the fluctuating surface wave amplitudes:

$$C_{\xi\xi}(k, \omega) = (1/\pi)^2 \{ (k_B T \nu k^3 / \rho) / [(\omega^2 - \omega_0^2)^2 + (2\nu k^2 \omega)^2] \} f(kl_c). \quad (11)$$

Equation (11) is similar to the well known classical result, the only difference being the presence of the function $f(kl_c)$. The classical expression could be considered as a limiting form of (11) in the case when $l_c = 0$ ($f(0) = 1$) corresponding to neglecting the viscous-dissipative spatial correlation.

The spectral density $C_{\xi\xi}(k, \omega)$ is a basic quantity in the description of light scattering from fluid interfaces [11-13]. The maximum in the frequency distribution experimentally observed is closely related to the values of the kinematic viscosity ν and the local surface tension γ . This could be easily explained by the type of $C_{\xi\xi}(k, \omega)$ defined in (11). A new result following from this expression is the dependence of the scattered light on the wavenumbers obtained by means of the function $f(kl_c)$. However, since the value of l_c is too small, this effect will be observed in the region of high wavevectors $k > l_c^{-1}$.

Another consequence of the results reported here is the mean square velocity of the surface wave amplitude, which is expressed by the spectral density as follows: $\langle (\partial\xi/\partial t)^2 \rangle = 2 \int_0^\infty \int_0^\infty \omega^2 C_{\xi\xi}(k, \omega) k dk d\omega$. After substituting (11) in this relation the result $\langle (\partial\xi/\partial t)^2 \rangle = (k_B T / 2\pi\rho) \int_0^\infty f(kl_c) k^2 dk = k_B T / \rho V_c$ is derived ($V_c \sim \pi l_c^3$), which is in accordance with the classical notion of the equipartition energy distribution [2].

Further considerations require the spatial spectral density of the autocorrelation function of the surface wave amplitudes $K_{\xi\xi}(k)$ to be obtained:

$$K_{\xi\xi}(k) = \int_{-\infty}^{\infty} C_{\xi\xi}(k, \omega) d\omega = \frac{k_B T f(kl_c)}{2\pi(\gamma k^2 + \rho g)}. \quad (12)$$

Equation (12) is close to the classical result obtained by Mandelstam [14]. The only difference is the presence of the function $f(kl_c)$. Since $f(0) = 1$, the thermodynamic sum rule $2\pi K_{\xi\xi}(0) = k_B T / \rho g$ follows from (12).

The mean square local amplitude is a very interesting characteristic of the fluctuating capillary-gravity surface waves. It is defined by $K_{\zeta\zeta}(k)$ as $\langle \zeta^2 \rangle = \int_0^\infty K_{\zeta\zeta}(k) k dk$ and substituting in (12); after some rearrangements, $\langle \zeta^2 \rangle$ is derived as

$$\langle \zeta^2 \rangle = \frac{k_B T}{4\pi\gamma} \int_0^\infty \frac{f(\sqrt{u})}{u + (l_c/l)^2} du = \frac{k_B T}{4\pi\gamma} F\left(\frac{l_c}{l}\right) \quad (13)$$

where the so-called capillary length $l = \sqrt{\gamma/\rho g}$ is introduced. From (13) and the properties of the function $f(\cdot)$ it follows that, in the case when the viscous-dissipative spatial correlations ($l_c = 0$) are neglected, $\langle \zeta^2 \rangle$ will tend to infinity [14, 15].

To avoid this contradiction between the classical spatial spectral density and the physical reality, Buff *et al* [15] introduced an effective upper limit k_m of the wavevectors (the so-called cut-off vector) and the mean square amplitude has the form

$$\langle \zeta^2 \rangle = \frac{k_B T}{2\pi} \int_0^{k_m} \frac{k}{\gamma k^2 + \rho g} dk = \frac{k_B T}{4\pi\gamma} \ln(1 + k_m^2 l^2). \quad (14)$$

As seen from (14), this procedure leads to a result similar to that of (13), but the value of k_m remains unclear. In the literature, k_m^{-1} is proposed to be proportional to the root mean square amplitude [15–17], the molecular diameter [6, 12, 18] and the intrinsic thickness of the interface [19]; other interpretations [20–22] are reported as well. However, none of these is connected to the existence of the spatial dissipative correlation, which is the basic element of the present theory.

According to (13), the model (14) corresponds to a special type of the function $f(kl_c) = H(k_m - k)$, where $H(\cdot)$ is a Heaviside function. The upper limit k_m of wavevectors should be determined by the relation $k_m l_c \sim 1$ (i.e. $k_m \nu \sim c$) and have clearly expressed dissipative character. This relation shows that waves with a wavelength smaller than the correlation length of the bulk stress tensor fluctuations are not generated. To estimate k_m it is necessary to take into account the following circumstances. Since k_m is in the region of high wavevectors, where the kinematic viscosity depends significantly on the wavenumber [1, 23] i.e. $\nu(k)$ (it is not necessary to consider the ω -dependence of ν , because the present theory is an approximation for low frequencies), to determine k_m one must use the relation $k_m \nu(k_m) \lesssim c$, where k_m is the lowest wavevector satisfying this relation. From this expression and the asymptote of the k -dependence of ν ($\nu(k) \rightarrow c/k$, $\forall k > \lambda^{-1}$, λ being the mean free path of molecules [23]) it can be concluded that $k_m \sim \lambda^{-1}$.

The mean free path of molecules in liquids is a rather complicated quantity [24]. For simple liquids an estimation of λ can be proposed: $\lambda = V/\pi d^2 G(d)$, where V is the volume per molecule, d is the molecular diameter and $G(\cdot)$ is the pair distribution function. Experimental and theoretical data show that the values of G are in the range 1–5 [24].

In table 1 the values of k_m experimentally measured by ellipsometry are presented. They are used to calculate λ and G by means of the relation between the cut-off vector and the mean free path of molecules ($k_m^{-1} = \lambda$) proposed in this work. The obtained estimates of G ($G = 1$ –7) are in good quantitative agreement with the values mentioned above, which shows that the cut-off vector predicted here is realistic.

In the literature [18] sometimes the behaviour of the mean square amplitude $\langle \zeta^2 \rangle$ in the limit $g \rightarrow 0$ is discussed. As follows from equations (13) and (14), $\langle \zeta^2 \rangle (g \rightarrow 0)$ tends to infinity. This effect is due to the infinite depth of the liquid. In the case of a finite depth the divergence of $K_{\zeta\zeta}(0)$ at $g \rightarrow 0$ disappears since a term accounting for the fluid compressibility should occur, leading to the well known thermodynamic limit $2\pi K_{\zeta\zeta}(0) = k_B T/(\rho g + \kappa/h)$, κ being the liquid compressibility and h the liquid depth.

Table 1. Calculated data of the pair distribution function G .

Substance	k_m^{-1} (Å)	M	ρ (kg m ⁻³)	d (Å)	G
From the work of Bouhet [25]					
Anethole	1.00	148	988	6.0	2.2
Bromonaphthalene	0.92	207	1489	4.9	3.3
Bromoform	0.93	253	2890	5.0	2.0
Benzene	0.77	78	879	5.0	2.4
<i>p</i> -xylene	0.74	106	860	5.6	2.8
<i>o</i> -xylene	0.72	106	897	5.6	2.8
Carbon tetrachloride	0.72	154	1594	5.2	2.7
Quiniline	0.89	129	1094	5.5	2.3
<i>m</i> -xylene	0.70	106	868	5.6	3.0
Cyclohexane	0.75	84	779	5.4	2.7
Anisole	0.77	108	995	5.4	2.6
Ethyl cinnimate	0.75	176	1049	6.2	3.1
Dibromyl ethane	0.80	188	2180	5.7	1.7
Benzyl benzoate	0.79	212	1112	6.5	3.0
1-nonanol	0.65	144	827	6.1	3.8
Ethyl acetate	0.64	88	900	5.2	3.0
Ethyl benzoate	0.67	150	1046	5.9	3.3
Ethanol	0.56	46	789	4.4	2.9
Oleic acid	0.63	282	895	7.7	4.5
Acetic acid	0.64	60	1049	4.3	2.5
Diethyl oxalate	0.66	146	1078	5.8	3.3
Nitrobenzene	0.59	123	1204	5.3	3.3
Glycerin	0.88	92	1261	4.7	2.0
Water	0.86	18	1000	3.0	1.2
From the work of Kizel [26]					
Pentane	0.38	72	626	5.5	5.4
Hexane	0.29	86	660	5.7	7.3
<i>n</i> -octane	0.49	114	704	6.1	4.7
Iso-octane	0.45	114	703	6.1	5.1
Methyl alcohol	0.31	32	793	3.9	4.6
Ethyl alcohol	0.58	46	789	4.4	2.8
<i>n</i> -propyl alcohol	0.52	60	804	4.7	3.4
Iso-propyl alcohol	0.54	60	785	4.8	3.3
<i>n</i> -butyl alcohol	0.62	74	810	5.1	3.0
<i>n</i> -amyl alcohol	0.63	88	814	5.4	3.2
Iso-amyl alcohol	0.58	88	812	5.4	3.5
Octyl alcohol	0.65	130	825	6.1	3.5
Formic acid	0.34	46	1226	3.8	4.1
Acetic acid	0.67	60	1049	4.3	2.4
Propionic acid	0.65	74	992	4.7	2.7
Butyric acid	0.60	88	959	5.1	3.1
Caproic acid	0.71	116	945	5.6	2.9
Pelargonic acid	0.65	158	905	6.3	3.6
Palmitic acid	0.68	256	850	7.6	4.1
From the work of Beaglehole [11, 19, 27]					
Cyclohexane	0.68	84	779	5.4	2.9
Methanol	0.60	32	789	4.0	2.3
Argon	1.01	40	1402	3.4	1.3
Carbon tetrachloride	0.82	154	1594	5.2	2.3
Water	1.08	18	1000	3.0	1.0

Another interesting statistical characteristic of the fluctuating capillary-gravity waves is the correlation area S_{ξ} of their amplitudes. A standard definition [2] $S_{\xi} = \int_0^{\infty} |K_{\xi\xi}(R)|/(\xi^2) 2\pi R dR$ can be used, where $K_{\xi\xi}(R)$ is the spatial correlation function, $K_{\xi\xi}(k)$ being its Fourier image. Owing to difficulties in the determination of the inverse Fourier image of the spectral density $K_{\xi\xi}(k)$ from equation (12) and the calculation of the integral, in the previous work [28] an alternative definition is introduced:

$$S_{\xi} = \int_0^{\infty} \left(\frac{K_{\xi\xi}(R)}{\xi^2} \right)^2 2\pi R dR = 2\pi \left(\int_0^{\infty} K_{\xi\xi}^2(k)k dk \right) / \left(\int_0^{\infty} K_{\xi\xi}(k)k dk \right)^2$$

leading to direct calculation of S_{ξ} from the spectral density.

It is seen from this equation that S_{ξ} depends on both the capillary length l and the correlation length l_c of the stress tensor fluctuations. At temperatures much below the critical temperature ($l \gg l_c$), S_{ξ} will be proportional to πl^2 (l plays the role of the non-dissipative correlation length related to conservative interactions), which is a well known result [18]. However, close to the critical point the capillary length tends to zero and S_{ξ} will be of the order of the bulk correlation area $S_c \sim \pi l_c^2$. This behaviour is to be expected since at the critical temperature a bulk homogeneous system is established.

In this manner the correlation area S_{ξ} of the surface wave amplitude velocities could be calculated:

$$S_{\xi} = \int_0^{\infty} \left(\frac{K_{\xi\xi}(R)}{\xi^2} \right)^2 2\pi R dR \sim \int_0^{\infty} (kl_c^3 f(kl_c))^2 2\pi k dk \sim S_c.$$

This result shows that the correlation length of the velocities is proportional to l_c , i.e. its viscous-dissipative character is confirmed once again.

A difference should be made between the local surface tension γ , measured by the light scattering method [13] for instance, and the macroscopic (macrogeometric) value γ_0 defined by $\gamma_0 = dW/(dx dy)$, where dW denotes the work of forming the interfacial surface. When the real situation at the interface is taken into account, this work can be expressed as $dW = [\gamma(1 + \frac{1}{2}((\text{grad } \xi)^2)) + \frac{1}{2}\rho g(\xi^2)] dx dy$. Here both the real value of the area formation and the work related to the gravity force are taken into account. The replacement of this equation in the definition of γ_0 leads to a link between γ and γ_0 : $\gamma_0 = \gamma + \frac{1}{2} \int_0^{\infty} (\gamma k^2 + \rho g) K_{\xi\xi}(k)k dk = \gamma + k_B T/2S_c$, where (12) is used. A similar relation expressed by k_m is obtained by Buff *et al* [15].

From the above considerations it follows that γ_0 is always higher than γ as a consequence of the increase in the real area in comparison with the macrogeometric area. The difference $\gamma_0 - \gamma$ is equal to the work per unit area related to the existence of the fluctuating capillary-gravity waves. According to the equipartition law, it is equal to the surface density $k_B T/2S_{\xi}$ of the kinetic energy of the fluctuating surface waves.

5. Concluding remarks

In the present paper the influence of the stress tensor fluctuations in the bulk on a liquid-gas interface is studied and, since they are spatially correlated because of the viscous-dissipative interactions [7, 8], finite surface wave characteristics such as mean square amplitude and energy density are established. In the literature the surface wave spatial correlations are treated by assuming an upper limit of wavevectors called a cut-off vector. It follows, from

the present theory, that the cut-off wavevector is directly related to the viscous-dissipative correlation length.

The shear viscous flow is neither the single nor the most important dissipative effect which could influence the cut-off vector. All transport phenomena involved in entropy production should contribute to l_c by their phenomenological transfer coefficients (in the case of a liquid-gas interface these are shear and dilatation viscosities and heat conductivity [2, 10]). Since k_m is in the region of high wavevectors and all transfer coefficients tend to c/k , $\forall k > \lambda^{-1}$ (the fluid particles can be considered almost free), the estimation $k_m \sim \lambda^{-1}$ will be general.

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